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Math 118 - Fall 2023 - Common Final Exam, version A

Print name:____

Section number:__________ **Instructor's name:**______________________________

Directions:

- This exam has 13 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to two decimal places if necessary.**

Good luck!

Question:	1	2	3	4	5	6	7
Points:	9	9	7	5	12	6	12
Score:							

Question:	8	9	10	11	12	13	Total
Points:	9	6	9	5	6	5	100
Score:							

1. (9 points) The output of Kaden's banana farm is 2500 bananas in the year 2023. Recall that a linear function has a general form of $P = mt + b$ and an exponential function has a general form of $P = a \cdot b^t$.

   (a) If Kaden increases his banana output by a rate of 200 bananas per year, find a formula for the function $P(t)$, the number of bananas t years after 2023.

   (b) If the output is decreasing by 8% per year, find a formula for the function $P(t)$, the number of bananas t years after 2023.

   (c) Under the assumptions stated in part b, find the year that banana output will hit $1,000$. Round to the nearest whole number.

2. (9 points) Oli opens a bank account with an initial deposit of \$7000. It earns interest at a nominal rate of 5% per year. Find the balance of their account after 6 years if interest is compounded as follows.

   (a) Annually (once a year).

   (b) Monthly (twelve times per year).

   (c) Continuously.

3. (7 points) Consider the exponential function $Q = 10.2(0.851)^t$.

   (a) Determine if this function displays exponential growth or decay.
   Circle one: **exponential growth** or **exponential decay**. Explain your answer in a sentence.

   (b) Give the initial value, growth factor, and growth rate for the given function.

   The initial value is __________

   The growth factor is __________

   The growth or decay rate is __________

   (c) Write the given function in the form $Q = ae^{kt}$.

4. (5 points) The chemistry department at Loyola University Chicago discovers a new element and names it "Jesuitinium." Jesuitinium decays at a continuous rate of 5% per hour. Find the half-life of Jesuitinium. Make sure to include units in your answer.

5. (12 points) The number of mice that live on the Ganshert family farm oscillates sinusoidally between a low of 1000 on January 1st ($t = 0$), and a high of 5000 on July 1st ($t = 6$).

(a) Find the amplitude, period, and midline of the function $P = f(t)$.

The amplitude is __________

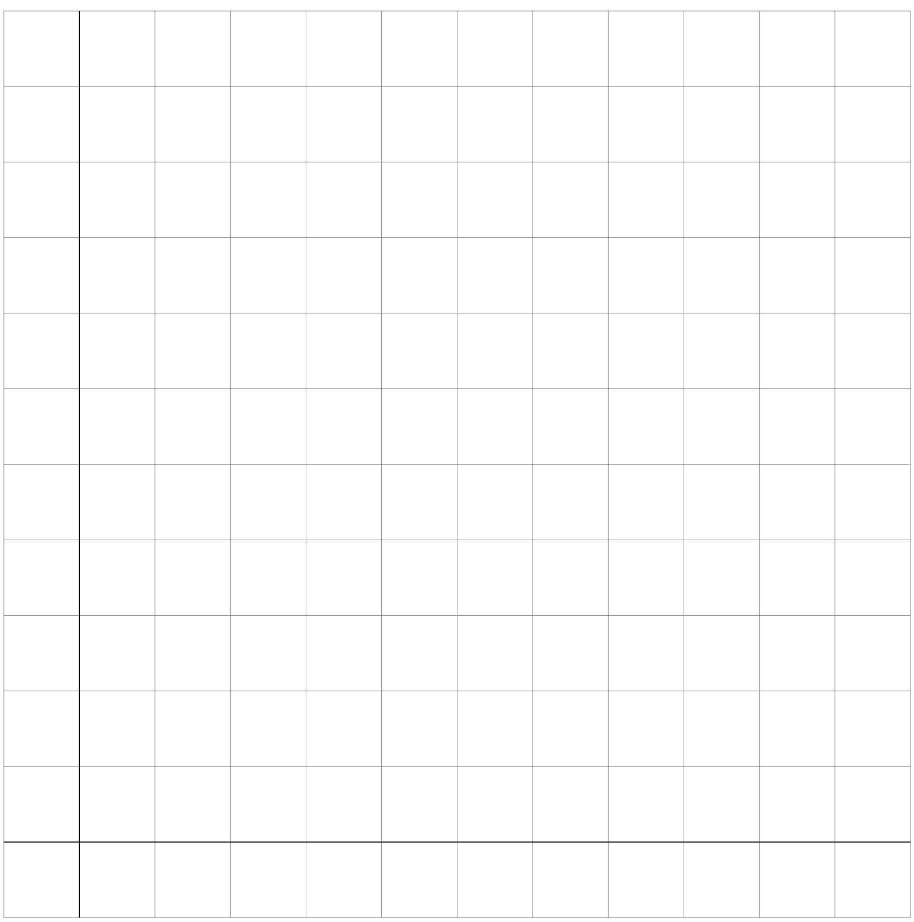
The period is __________

The midline is __________

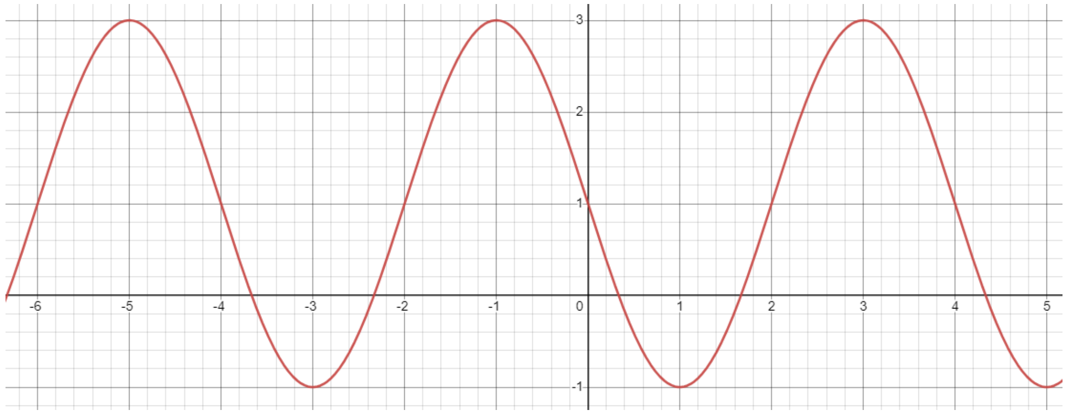
(b) Find a formula for the population, P, in terms of time, t, in months since January 1st.

(c) Write an equation for the first time that the number of mice that live on the farm is 3500. Find a solution to this equation, giving your answer in terms of an inverse trig function and also evaluate with correct units

(d) Graph P as a function of t.



6. (6 points) Find a formula of the trigonometric function shown in the graph below.



7. (12 points) For angles α and β such that $\frac{\pi}{2} < \alpha < \pi$ and $0 < \beta < \frac{\pi}{2}$ such that $\sin(\alpha) = \frac{4}{5}$ and $\cos(\beta) = \frac{3}{8}$, find the given quantities without finding α and β. Give an exact answer for each part.

(a) $\cos(\alpha)$

(b) $\sin(\beta)$

(c) $\sin(\alpha + \beta)$

(d) $\cos(\alpha + \beta)$

8. (9 points) A ladder is leaning against a building. The base of the ladder is 5 meters from the base of the building, and the ladder forms a 37° angle with the ground. The top of the ladder is exactly at the top of the building.

   (a) Draw a picture of this situation.

   (b) Find the height of the building.

   (c) Find the length of the ladder.

9. (6 points) Let $f(x) = 5x - 3$, $g(x) = 2x + 7$ and $h(x) = \log(x)$. Find the following, and simplify your answers completely:

   (a) $g(f(3))$

   (b) $h(f(g(x)))$

10. (9 points) Let $P = f(t) = 300(1.182)^t$ be the number of people in the United States that have caught a new disease known as "Mathitis." Let t be measured in years since 2023.

(a) Evaluate $f(4)$. Round to the nearest whole number. Describe in words what this quantity represents. Write your answer in a complete sentence with units.

(b) Find a formula for $f^{-1}(P)$ in terms of P. Give an exact answer.

(c) Evaluate $f^{-1}(1500)$. Round to the nearest whole number

(d) Describe in words what the quantity you found in part c) represents. Write your answer in a complete sentence with units.

11. (5 points) Decompose the function

$$f(x) = \ln(15x - 3)$$

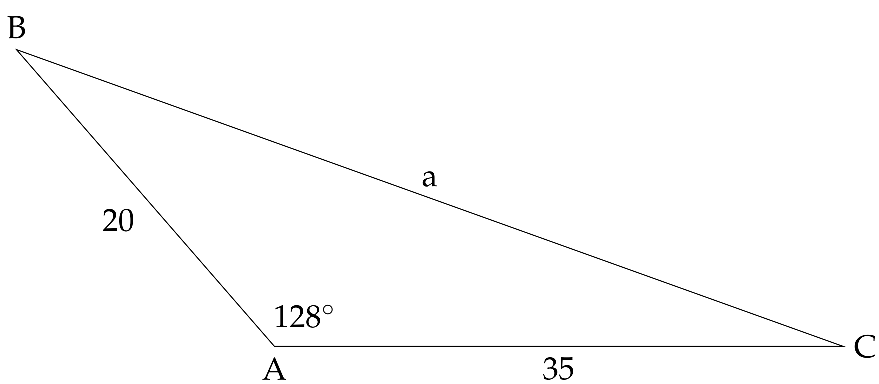
into a composition of two new functions u and v, where v is the inside function. That is $f(x) = u(v(x))$, so that $u(x) \neq x$ and $v(x) \neq x$.

12. (6 points) Perform the following conversions.

(a) Convert the Cartesian coordinates $(8, 8)$ to polar coordinates. Give an exact answer.

(b) Convert the polar coordinates $(2, \frac{\pi}{3})$ to Cartesian coordinates. Give an exact answer.

13. (5 points) Two planes fly from a point A. The angle between their two flight paths is 128 degrees. One plane has flown 20 miles from point A to point B. The other plane has flown 35 miles from point A to point C. How far apart are the two planes? A diagram is below



Exponential and Logarithm Formulas
Linear Function: $Q(t) = mt + b$
Exponential Function: $Q(t) = a \cdot b^t$
Continuous Exponential Function: $Q(t) = a \cdot e^{kt}$
Simple Interest: $B = P(1 + r)^t$
Compound Interest: $B = P\left(1 + \frac{r}{n}\right)^{nt}$

Trigonometry Formulas
1 radian $= \frac{180}{\pi}$ degrees and 1 degree $= \frac{\pi}{180}$ radians

$$\sin(\theta) = \frac{opp}{hyp} = \frac{y}{r} \quad \cos(\theta) = \frac{adj}{hyp} = \frac{x}{r} \quad \tan(\theta) = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y} \quad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x} \quad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities: $\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta) \quad 1 + \cot^2(\theta) = \csc^2(\theta)$

Sum and Difference Formulas:
$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

Even-Odd Identities: $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$ and $\tan(-x) = -\tan(x)$

Other identities: $\sin(\theta) = \sin(\pi - \theta)$, $\cos(\theta) = -\cos(\pi - \theta)$ and $\tan(\theta) = -\tan(\pi - \theta)$

General form for sine and cosine: $f(t) = A\sin(Bt) + k$ and $f(t) = A\cos(Bt) + k$

General form with horizontal shift: $f(t) = A\sin(B(t - h)) + k$ and $f(t) = A\cos(B(t - h)) + k$)

Period for sine and cosine: $P = \frac{2\pi}{|B|}$ or $PB = 2\pi$

Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Arc Length: $s = r\theta$

Inverse Trig Functions
$\theta = \cos^{-1}(y)$ provided that $y = \cos(\theta)$ and $0 \leq \theta \leq \pi$
$\theta = \sin^{-1}(y)$ provided that $y = \sin(\theta)$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
$\theta = \tan^{-1}(y)$ provided that $y = \tan(\theta)$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Polar coordinates conversions
$r^2 = x^2 + y^2$, $\tan(\theta) = \frac{y}{x}$, $x = r\cos(\theta)$, $y = r\sin(\theta)$

The Unit Circle